Firm Dynamic Hedging and the Labor Risk Premium

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PRELIMINARY AND INCOMPLETE

Risk Premium View of Business Cycles

- Recessions are financial panics in the labor market

$$w_t = \mathbb{E}_{it}\left[z_{it}\right] \times \left(1 + \underbrace{Cov_{it}(\hat{m}_{it+1}; \hat{z}_{it})}_{-\pi_{it}}\right)$$

- A heterogenous-agent model of pricing kernel m_{it+1} and marginal product of labor z_{it}, emphasizing the persistence of uninsurable idiosyncratic shocks:
  - dynamic hedging motive for the labor risk premium
  - heterogenous risk premiums $\implies$ endogenous fluctuations in TFP
  - aggregate risk sharing: financial amplification channel

- Use firm-level data to quantitatively evaluate model

Setting

- Entrepreneurs and representative worker:

$$U_w(c_w, \ell) = \mathbb{E}\left[\sum \beta_w^t \left(\frac{c_{wt}^{1-\gamma}}{1-\gamma} - \frac{\ell_t^{1+1/\psi}}{1+1/\psi}\right)\right]$$

$$U_e(c_i) = \mathbb{E}\left[\sum \beta_e^t \left(\frac{c_{it}^{1-\gamma}}{1-\gamma}\right)\right]$$

- Entrepreneurs hire labor to produce

$$y_{it} = z_{it}\ell_{it} \tag{1}$$

- Aggregate resource constraints

$$c_t = \int c_{it} di + c_{wt} = \int y_{it} di \tag{2}$$

$$\ell_t = \int \ell_{it} di \tag{3}$$

Technology

- Productions is risky: $y_{it} = z_{it}\ell_{it}$

$$\log z_{it} = \rho\epsilon_{it-1} + \sigma_{ut}u_{it} - \frac{1}{2}\rho^2\frac{\bar{\sigma}_\eta^2}{1-\rho^2} - \frac{1}{2}\sigma_{ut}^2$$

$$\epsilon_{it} = \rho\epsilon_{it-1} + \bar{\sigma}_\eta\eta_{it}$$

$$\begin{bmatrix} u_{it} \\ \eta_{it} \end{bmatrix} \sim N\left(0, \begin{bmatrix} 1 & \lambda \\ \lambda & 1 \end{bmatrix}\right)$$

- Time-varying labor productivity risk

$$\sigma_{ut} = \rho_\sigma\sigma_{ut-1} + v_t$$

- Distribution of conditional expectations is invariant:

$$\mathbb{E}_{it}\left[z_{it}\right] = e^{\rho\epsilon_{it-1} - \frac{1}{2}\rho^2\frac{\bar{\sigma}_\eta^2}{1-\rho^2}} = \bar{z}_{it} \sim LogN(-\frac{1}{2}\rho^2\frac{\bar{\sigma}_\eta^2}{1-\rho^2}, \rho^2\frac{\bar{\sigma}_\eta^2}{1-\rho^2})$$

$$\mathbb{E}\left[z_{it}\right] = 1$$

$\Longrightarrow$ all effects are driven by risk premia

Timing, information, and agents' problems

- Each period t:
  1. **Aggregate** state $s_t = \sigma_{ut}$ realized
  2. **Consumption and labor markets**: workers and entrepreneurs choose consumption and labor supply/demand: (c_{wt}, ℓ_t) and $(c_{it}, \ell_{it})_{i \in [0,1]}$.
  3. **Production and trading**: idiosyncratic shocks $h_{it} = (u_{it}, \eta_{it})$ revealed; production and consumption takes place; Arrow securities pay and new Arrow securities for next period $n_{it+1}(s')$ are traded with price $q_t(s')$.

- Entrepreneurs choose $(c_{it}(s^t, h^{t-1}), \ell_{it}(s^t, h^{t-1}), n_{it+1}(s'; s^t, h^t))$ subject to

$$\int q_t(s') n_{it+1}(s') ds' = n_{it} - c_{it} + (z_{it} - w_t)\ell_{it}$$

- Workers choose $(c_{wt}(s^t), \ell_t(s^t), n_{wt+1}(s'; s^t))$ subject to

$$\int q_t(s') n_{wt+1}(s') ds' = n_{wt} - c_{wt} + w_t \ell_t$$

Competitive equilibrium

- Some equilibrium conditions are standard:
  - Euler equations
$$c_{wt}^{-\gamma} = \beta_w(1+r_t)\mathbb{E}_t\left[c_{wt+1}^{-\gamma}\right]$$
$$c_{it}^{-\gamma} = \beta_e(1+r_t)\mathbb{E}_{it}\left[c_{it+1}^{-\gamma}\right]$$
  - Aggregate risk sharing
$$\frac{c_{wt+1}(s^t, s_{t+1})}{c_{wt+1}(s^t, s'_{t+1})} = \frac{c_{it+1}(s^t, s_{t+1}, h^t)}{c_{it+1}(s^t, s'_{t+1}, h^t)} = \frac{c_{t+1}(s^t, s_{t+1})}{c_{t+1}(s^t, s'_{t+1})}$$
  - Labor supply
$$\ell_t^{1/\psi} = c_{wt}^{-\gamma} w_t$$
- The novel part is labor demand

Labor demand

- Homothetic pref. + linear technology:

$$V_t(s^t, h^{t-1}) = V_t(n, \epsilon_-; s^t) = \frac{\left(A(\epsilon_-; s^t)n\right)^{1-\gamma}}{1-\gamma}$$

- Labor demand

$$\mathbb{E}\left[\overbrace{A(\epsilon_{it}; s^{t+1})^{1-\gamma} \times n'(s_{t+1}; u_{it})^{-\gamma}}^{m_{it+1}=\partial_n V_{t+1}} \times (z_{it} - w_t) \,|s^t, \epsilon_{it-1}\right] = 0$$

or re-arranging:

$$w_t = \mathbb{E}_{it}\left[z_{it}\right] \times \left(1 + \underbrace{Cov_{it}(\hat{m}_{it+1}; \hat{z}_{it})}_{-\pi_{it}}\right)$$

The labor risk premium π_{it} acts like a labor wedge

Understanding labor demand: labor is risky

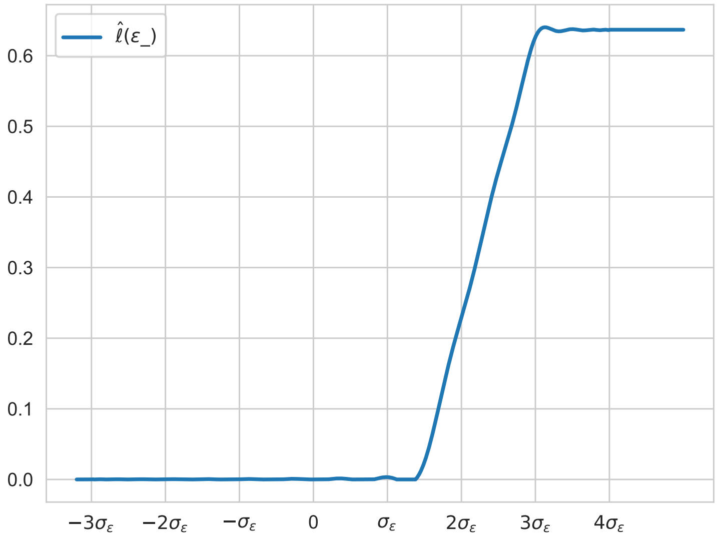
- In steady state:

$$\mathbb{E}\left[\overbrace{A(\epsilon_{it};s^{t+1})^{1-\gamma}\times\left(1+\hat{\ell}_{it}(z_{it}-w_t)\right)^{-\gamma}}^{m_{it+1}}\times(z_{it}-w_t)\,\middle|s^t,\epsilon_{it-1}\right]=0$$

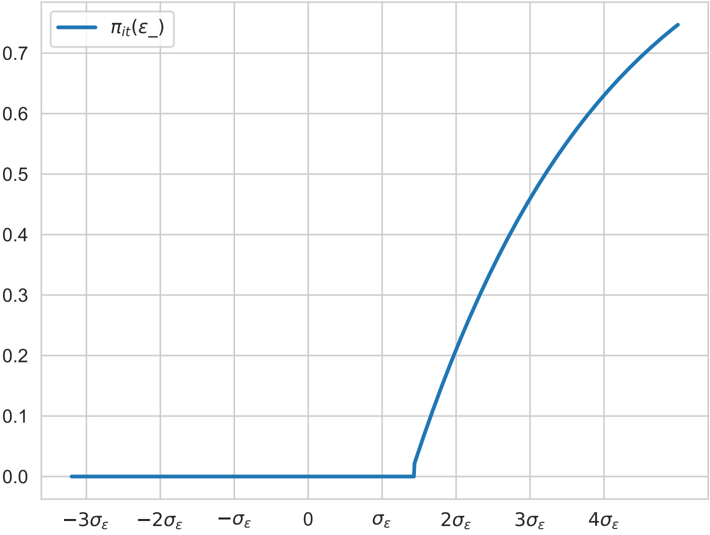
  where $\hat{\ell}_{it}$ is the portfolio-weight on labor (labor/ net worth)

- Larger $\hat{\ell}_{it} \implies$ stronger covariance of marginal product of labor with individual SDF

More productive firms hire more labor



Risk premium π_{it} is larger for more productive firms



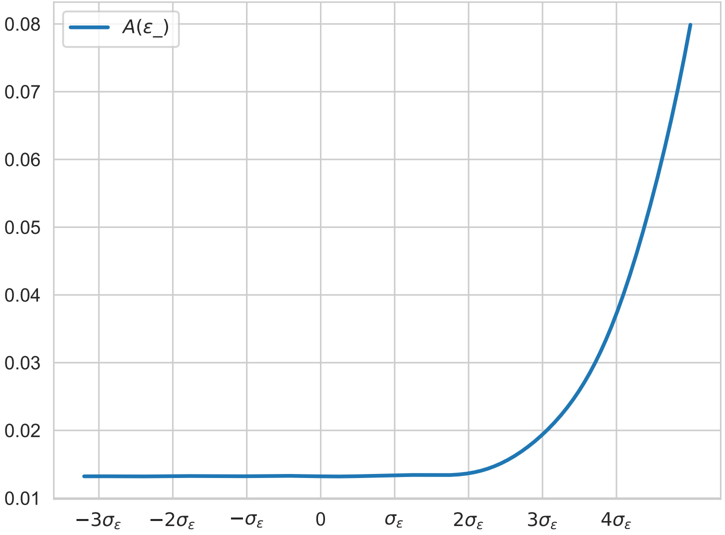
Persistent idiosyncratic shocks and dynamic hedging

- Labor demand

$$\mathbb{E}_{it}\left[\overbrace{A_{it+1}^{1-\gamma}(\eta_{it}) \times n_{it+1}^{-\gamma}(u_{it};\hat{\ell}_{it})}^{m_{it+1}} \times (z_{it}(u_{it}) - w_t)\right] = 0$$

  - Persistence of idiosyncratic productivity shocks $\lambda_{it} = Corr_{it}(u_{it}, \eta_{it})$
  - Key economics: income vs. substitution effects
  - Special case $\gamma = 1$: myopic optimization

High productivity $\implies$ better investment opportunities



Dynamic hedging

- Investment opportunities $A(\epsilon_-)$ are increasing in ϵ_-
- If $\lambda > 0$ the marginal product of labor z_{it} is positively correlated with investment opportunities A_{it+1}
- With $\gamma > 1$, this means a negative correlation with the entrepreneurs SDF

Aggregate risk sharing and state variables

- From aggregate risk sharing, we know the consumption share

$$\theta_t(\epsilon_-; s^{t-1}) := \frac{\int_{\{\epsilon_{it-1}=\epsilon_-\}} c_{it} di}{c_t}, \qquad \theta_{wt} = 1 - \int \theta_t(\epsilon_-) d\epsilon_-$$

  is predetermined (does not respond on impact to aggregate shocks). We can derive a law of motion for $\theta_t(\epsilon_-)$

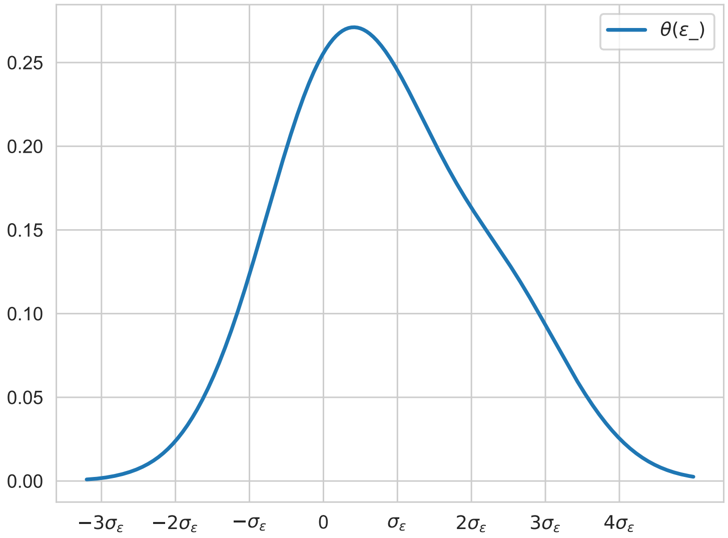
- Notice aggregate wealth by productivity type is not predetermined:

$$\omega_t(\epsilon_-; s^t) := \int_{\{\epsilon_{it-1}=\epsilon_-\}} n_{it} di = \theta_t(\epsilon_-; s^{t-1}) \times A_t(\epsilon_-; s^t)^{\frac{1-\gamma}{\gamma}} \times c_t$$

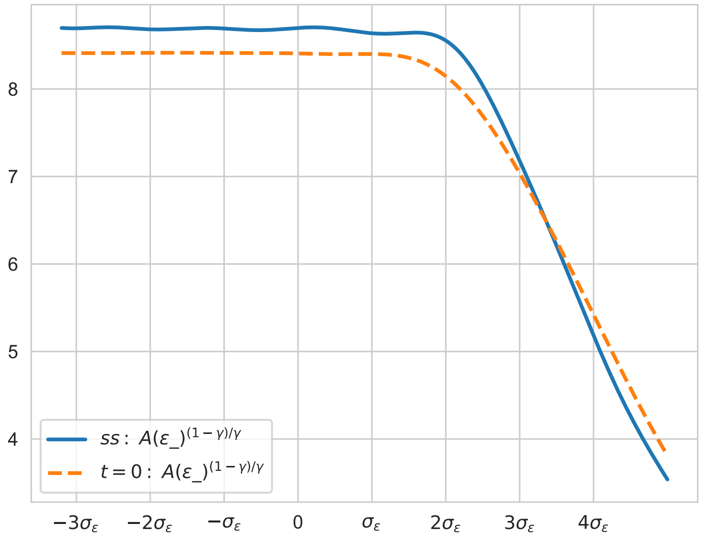
  $\gamma \neq 1 \implies$ financial amplification channel through $A^{\frac{1-\gamma}{\gamma}}$ in response to tradeable aggregate shocks (for $\gamma = 1$ we have $A_t(\epsilon_-)^{\frac{1-\gamma}{\gamma}} = \frac{1}{1-\beta_e}$)

- State variables: σ_{ut} (exogenous) and $\theta_t(\epsilon_-)$ (endogenous but "slow moving")

Consumption shares $\theta(\epsilon\_)$



An increase in σ_u redistributes wealth



Aggregate employment and output: iid shocks

- Assume $\epsilon_{it} = 0$ always $\implies$ uniform expected productivity and risk premium: $\mathbb{E}_{it}[z_{it}] = Z = 1$ and $\pi_{it} = \pi_t$

$$w_t = Z \times \Big(1 - \pi_t\Big)$$

$$\pi_t = -Cov_t \left(\frac{(1 + \hat{\ell}_t(z_{it} - w_t))^{-\gamma}}{\mathbb{E}_t\left[(1 + \hat{\ell}_t(z_{it} - w_t))^{-\gamma}\right]} ; \frac{z_{it}}{\mathbb{E}_t[z_{it}]} \right)$$

- Plugging into labor supply equation:

$$\ell_t^{1/\psi + \gamma} = \theta_{wt}^{-\gamma} \times Z^{1-\gamma} \times (1 - \pi_t)$$

- Di Tella and Hall (2021): Higher risk $\sigma_{ut} \implies$ higher risk premium π_t (labor wedge) $\implies$ recession

Persistent shocks: labor risk premium and TFP

- Labor

$$\ell_t^{1/\psi+\gamma} = \theta_{wt}^{-\gamma} \times Z_t^{1-\gamma} \times (1 - \bar{\pi}_t)$$

where $\bar{\pi}_t$ is the production-weighted risk premium:

$$1 - \bar{\pi}_t = \int (1 - \pi_t(\epsilon_-)) \times \overbrace{\left(\mathbb{E}_t[z_{it}|\epsilon_-] \times \theta_t(\epsilon_-)(A_t(\epsilon_-)^{\frac{1-\gamma}{\gamma}} - 1)\hat{\ell}_t(\epsilon_-) \right)}^{y_t(\epsilon_-)/y_t} d\epsilon_-$$

and TFP:

$$Z_t = \frac{y_t}{\ell_t} = \left(\int \theta_t(\epsilon_-)(A_t(\epsilon_-)^{\frac{1-\gamma}{\gamma}} - 1)\hat{\ell}_t(\epsilon_-) d\epsilon_- \right)^{-1}$$

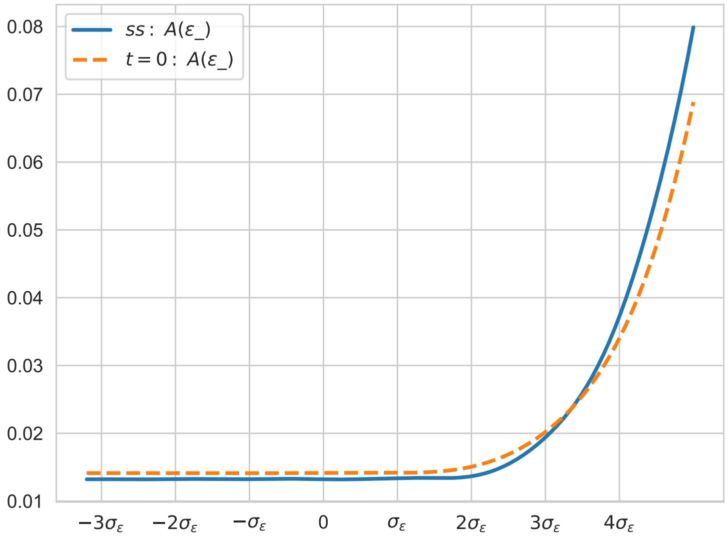
Risk premiums affect TFP and labor wedge

- TFP: heterogeneous risk premiums $\implies$ misallocation
  - fluctuations in risk premiums $\implies$ fluctuations in TFP

- Labor wedge: high productivity firms have higher risk premiums and matter more for the aggregate labor wedge

- Financial amplification channel: financial losses are distributed heterogeneously across productivity-types

Higher σ_u affects idiosyncratic dynamic hedging



Numerical solution

- We solve the model to first order in aggregate shocks using the sequence space (Bardoczy et al. 2021), with projection methods for the cross section.

Parameters	
γ	2
ψ	2
β_w	0.98
β_e	0.8
σ_u^{ss}	0.54
σ_η	0.15
λ	0.7
ρ	0.7
ρ_σ	0.5

Impulse response with persistent shocks

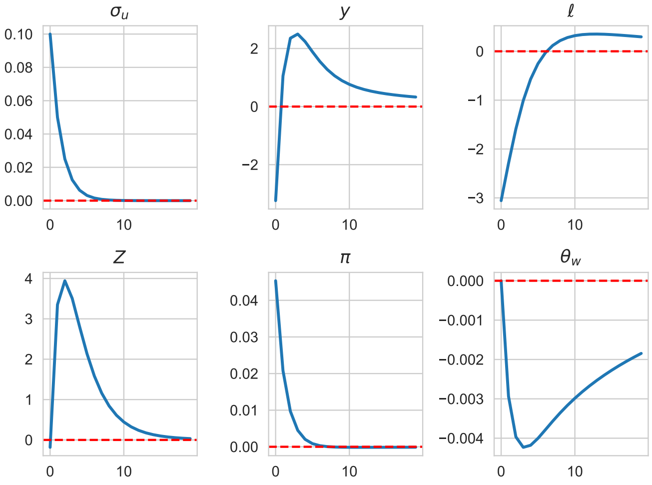
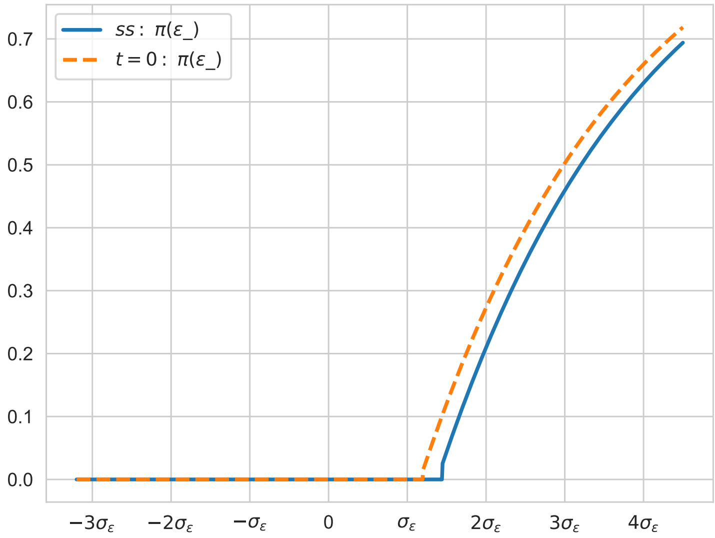


Figure: IRF to an increase in σ_u for output y, employment ℓ, TFP Z, labor risk premium π, and workers consumption share θ_w.

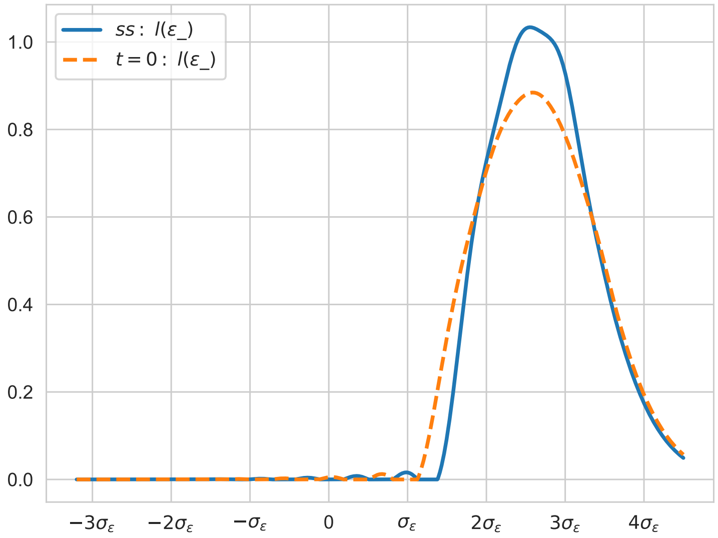
Impulse response with persistent shocks

- The labor risk premium π (labor wedge) spikes on impact and causes a contraction in employment ℓ and output y
- TFP Z falls initially, but subsequently rises above steady state, so output recovers much faster than employment, and overshoots steady state
- Workers consumption share θ_w does not respond on impact. It subsequently falls a little, but plays a quantitatively secondary role

Labor risk premiums



Employment allocation



Quantitative evaluation: work in progress

- Use data from Amadeus to construct firm-level productivity z_t following Bloom et al. (2019)

- Use GMM to estimate stochastic process for z_{it} and σ_{ut}

- TFPQ vs. TFPR
  - typically one can use labor to back out TFPQ (Klenow and Hsieh (2009))
  - here firms have heterogenous discounts: estimate TFPQ using full model

Conclusions

- A heterogenous-agent model of business cycles driven by labor risk premiums

- Persistent shocks $\implies$ dynamic hedging uninsurable idiosyncratic risk

- Heterogeneous risk premiums $\implies$ endogenous fluctuations in TFP