



# Past Automation and Future AI: How Weak Links Tame the Growth Explosion

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## How Much of Economic Growth is due to Automation?

- Automation has been going on since the Industrial Revolution
  - Human with hoes, to horses with plows, to tractors, to GPS automation
  - Robots replacing welders and painters in auto factories
  - AI is the latest form of automation, e.g., writing software
- How much of historical TFP growth was due to automation?
- What is the future of automation and growth?

## How We Answer these Questions

- Task-based production model — weak links ( $\sigma < 1$ )
- Standard BEA-BLS Industry Production Accounts data
- One heroic measurement — the automation rate
  - Historically, around 2% of not-yet-automated tasks get automated each year
  - Measure using ChatGPT & Claude
- For future, a task-based endogenous growth model  
Different cases:
  - AI continues past automation trends
  - Moore's Law everywhere – this time is different

## Main Results: Quantify the Impact of Automation on TFP Growth

- Historical accounting — automation generates most TFP growth
  - Switched from slowly-improving labor to rapidly-improving machines on ↑ tasks
  - Machines got better rapidly: 3.8pp faster than the rate at which humans got better
  - Majority of historical growth is from rapid improvement in machine productivity
- The future of A.I. — weak links tame the growth explosion
  - Calibrate an idea-based growth model to our historical facts
  - Exploding growth is possible but slowed by weak links
  - The path looks very similar over the next 20 years regardless of whether the capital share ends at 100% or 0% (LR future is very different)

## Related Literature

- Acemoglu and Restrepo (2017, 2018, 2019, 2020, 2021, 2022, 2023)
- Zeira (1998), Hemous and Olson (2016, 2025)
- Aghion, Jones, and Jones (2019)
- B. Jones and Liu (2024), B. Jones (2025), Caunedo and Keller (2024)
- Davidson (2021), Korinek and Suh (2024), Davidson, Halperin, Houlden, and Korinek (2025)

*Key contribution: Measurement and quantification*



## The Model

## Economic Environment: The Canonical Task Model

$$Y_t = Z_t \left( \int_0^1 \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \quad (1)$$

$$Y_{it} = \tilde{\psi}_{kit} K_{it} + \tilde{\psi}_{lit} L_{it} \quad (2)$$

$$K_t = \int_0^1 K_{it} di \quad \text{and} \quad L_t = \int_0^1 L_{it} di \quad \text{where } Z_t, \tilde{\psi}_{kit}, \tilde{\psi}_{lit}, K_t, L_t \text{ exogenous} \quad (3)$$

## Economic Environment: The Canonical Task Model

Complementarity  $Y_t = Z_t \left( \int_0^1 \alpha_i Y_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$  where  $\sigma < 1$  (1)

Substitution  $Y_{it} = \tilde{\psi}_{kit} K_{it} + \tilde{\psi}_{lit} L_{it}$  (2)

$K_t = \int_0^1 K_{it} di$  and  $L_t = \int_0^1 L_{it} di$  where  $Z_t, \tilde{\psi}_{kit}, \tilde{\psi}_{lit}, K_t, L_t$  exogenous (3)

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- New “procedures” as a way of incorporating new tasks

$$Y_{it} = \tilde{\psi}_{kit}^1 K_{it}^1 + \dots + \tilde{\psi}_{kit}^{N_{kt}} K_{it}^{N_{kt}} + \tilde{\psi}_{lit}^1 L_{it}^1 + \dots + \tilde{\psi}_{lit}^{N_{lt}} L_{it}^{N_{lt}}$$

- Notation:  $\psi_{kit} = \alpha_i^{\frac{\sigma}{\sigma-1}} \tilde{\psi}_{kit}$  and  $\psi_{lit} = \alpha_i^{\frac{\sigma}{\sigma-1}} \tilde{\psi}_{lit}$

## Representative Firm in a Perfectly Competitive Sector

$$\max_{\{K_{it}, L_{it}\}} P_t Y_t - w_t \int_0^1 L_{it} di - r_t \int_0^1 K_{it} di \quad \text{s.t. (1) + (2) given } P_t, w_t, r_t$$

- Use capital to produce task  $i$  whenever

$$\frac{\psi_{kit}}{r_t} \geq \frac{\psi_{lit}}{w_t}$$

- Define the set of tasks using capital and labor as

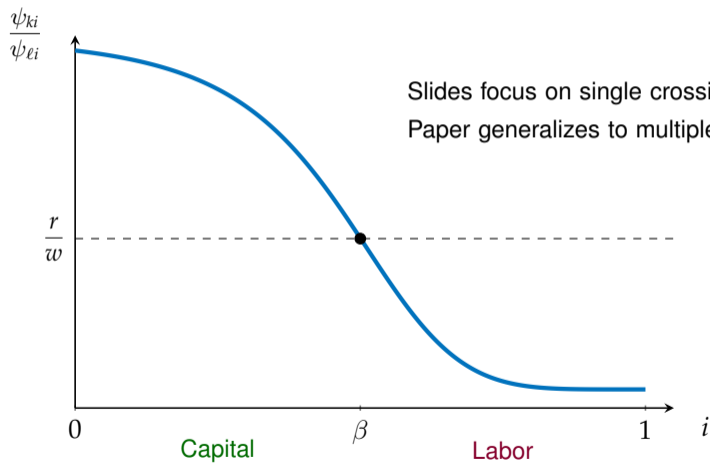
$$\Omega_{kt} = \{i \mid \psi_{kit}/\psi_{lit} \geq r_t/w_t\} \quad \text{and} \quad \Omega_{lt} = \{i \mid \psi_{kit}/\psi_{lit} < r_t/w_t\}$$

$$\beta_t \equiv \|\Omega_{kt}\| \quad \text{and} \quad 1 - \beta_t \equiv \|\Omega_{lt}\|$$

- Any task  $\beta$  that is just at the margin of being automated:

$$\frac{\psi_{k\beta t}}{\psi_{l\beta t}} = \frac{r_t}{w_t}$$

## Automation and Comparative Advantage: Example



Slides focus on single crossing case.  
Paper generalizes to multiple crossings.

## Proposition 1 (*Reduced-form production function*)

Equilibrium output  $Y_t$  can be represented as a CES-like production function:

$$\begin{aligned} Y_t &= F(B_t K_t, A_t L_t) \\ &= \left( (B_t K_t)^{\frac{\sigma-1}{\sigma}} + (A_t L_t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where

$$\begin{aligned} B_t &= Z_t \left( \int_{\Omega_{kt}} \psi_{kit}^{\sigma-1} di \right)^{\frac{1}{\sigma-1}} \\ A_t &= Z_t \left( \int_{\Omega_{\ell t}} \psi_{\ell it}^{\sigma-1} di \right)^{\frac{1}{\sigma-1}} \end{aligned}$$

## Proposition 1 (continued)

Factor shares are given by

$$s_{Kit} \equiv \frac{r_t K_{it}}{P_t Y_t} = \left( \frac{\psi_{kit} Z_t K_{it}}{Y_t} \right)^{\frac{\sigma-1}{\sigma}}$$

$$s_{Lit} \equiv \frac{w_t L_{it}}{P_t Y_t} = \left( \frac{\psi_{lit} Z_t L_{it}}{Y_t} \right)^{\frac{\sigma-1}{\sigma}}$$

$$s_{Kt} \equiv \frac{r_t K_t}{P_t Y_t} = \left( \frac{B_t K_t}{Y_t} \right)^{\frac{\sigma-1}{\sigma}} = \int_{\Omega_{kt}} s_{Kit} di$$

$\uparrow B \Rightarrow \downarrow s_K$

$$s_{Lt} \equiv \frac{w_t L_t}{P_t Y_t} = \left( \frac{A_t L_t}{Y_t} \right)^{\frac{\sigma-1}{\sigma}} = \int_{\Omega_{lt}} s_{Lit} di$$

*Task model has a familiar CES-like representation.*

*Not truly CES in that the  $A_t$  and  $B_t$  depend on  $w_t/r_t$  via  $\Omega_{kt}$  and  $\Omega_{lt}$ .*

## The Weights, $\omega_{kit}$ and $\omega_{lit}$

- Weights  $\omega_{kit}$  and  $\omega_{lit}$  are the shares of capital and labor costs for the tasks:

$$\frac{L_{it}}{L_t} = \frac{w_t L_{it}}{w_t L_t} = \frac{\psi_{lit}^{\sigma-1}}{\int_{\Omega_{\ell t}} \psi_{lit}^{\sigma-1} di} \equiv \omega_{lit}$$

$$\frac{K_{it}}{K_t} = \frac{r_t K_{it}}{r_t K_t} = \frac{\psi_{kit}^{\sigma-1}}{\int_{\Omega_{kt}} \psi_{kit}^{\sigma-1} di} \equiv \omega_{kit}$$

## Proposition 2 (*Growth rates of $B_t$ and $A_t$* )

Assuming tasks only get automated and never “de-automated,”

$$\hat{B}_t = \hat{Z}_t + \hat{\psi}_{kt} - \frac{1}{1-\sigma} \omega_{k\beta t} \dot{\beta}_t \quad \text{where} \quad \hat{\psi}_{kt} \equiv \int_{\Omega_{kt}} \hat{\psi}_{kit} \omega_{kit} di$$

Capital depleting

$$\hat{A}_t = \hat{Z}_t + \hat{\psi}_{\ell t} + \frac{1}{1-\sigma} \omega_{\ell\beta t} \dot{\beta}_t \quad \text{where} \quad \hat{\psi}_{\ell t} \equiv \int_{\Omega_{\ell t}} \hat{\psi}_{\ell it} \omega_{\ell it} di$$

Labor augmenting

- Effect of  $\uparrow \beta_t$  :
  - Capital depleting ( $\downarrow B$ ):  $K$  spread over **more** tasks — weakens each link
  - Labor augmenting ( $\uparrow A$ ):  $L$  concentrated on **fewer** tasks — strengthens links

*(see Aghion, Jones, and Jones 2019 for homogenous case)*

## Data and Sources: $Y/L$ , $TFP$ , and $s_K$

NAICS	Sector	Years
—	Private business sector	1950–2023
—	Agriculture	1950–2021
334	Computer and electronic products	1987–2017
3361–63	Motor vehicles, bodies and trailers, and parts	1987–2017
44–45	Retail trade	1987–2017
511, 516	Publishing industries (includes software)	1987–2017

- Private business: BLS Multifactor Productivity database (1950-2023)
- Agriculture: USDA Economic Research Service (1950-2021)
- BEA-BLS KLEMS Integrated Industry Production Accounts (1987-2021)

## Identification of $B_t$ and $A_t$

- Baseline:  $\sigma = 0.2$ 
  - Task elasticity  $\sigma$  would be the same as the EoS between  $K$  and  $L$  if the set of tasks being automated is held fixed
  - Since automation can adjust,  $\sigma$  is less than the estimated EoS(K,L)
  - EoS(K,L) estimated to be 0.4-0.5 by Young (2025) and 0.3 in meta analysis of Gechert et al (2022).
- Using model:
  - $y_t = Y_t/L_t$
  - $A_t = s_{L_t}^{\frac{\sigma}{\sigma-1}} y_t$
  - $\hat{Y}_t = s_{K_t} (\hat{B}_t + \hat{K}_t) + s_{L_t} (\hat{A}_t + \hat{L}_t)$
  - $T\hat{F}P_t := \hat{Y}_t - s_{K_t}\hat{K}_t - s_{L_t}\hat{L}_t = s_{K_t}\hat{B}_t + s_{L_t}\hat{A}_t$  implies  $B_t$

## TFP Growth: Basic Data

Sector	TFP growth	Growth in $B_t$	Growth in $A_t$	Factor share of capital	Factor share of labor
Private business	1.2	-1.0	2.3	0.35	0.65
Agriculture	3.3	2.6	4.3	0.57	0.43
Computers	12.8	9.8	14.9	0.41	0.59
Motor vehicles	1.7	0.4	2.7	0.43	0.57
Retail trade	1.7	-1.3	2.4	0.20	0.80
Software	1.8	-1.2	4.6	0.47	0.53

- Private business: standard TFP growth and factor shares
- $\hat{A}_t = 2.3\%$  while  $\hat{B}_t = -1.0\%$  — negative!
- $1.2 = .35 \cdot (-1.0) + .65 \cdot 2.3$ . Invariant to  $\sigma$  if factor shares are stable
- $\hat{B}_t$  is negative / tiny in 4 of the 6 sectors. Supports  $\sigma < 1$  and  $\uparrow \beta_t$

## TFP Growth: $\psi_{kit}$ , $\psi_{lit}$ , and $\beta_t$

- Using Proposition 2:

$$\begin{aligned}\widehat{TFP}_t &= s_{Kt}\hat{B}_t + s_{Lt}\hat{A}_t \\ &= \underbrace{s_{Kt}\hat{\psi}_{kt}}_{\text{Better capital}} + \underbrace{s_{Lt}\hat{\psi}_{lt}}_{\text{Better labor}} + \underbrace{\frac{\dot{\beta}_t}{1-\sigma}(s_{Lt}\bar{\omega}_{l\beta t} - s_{Kt}\bar{\omega}_{k\beta t})}_{\text{Automation effect}} + \hat{Z}_t\end{aligned}\quad (4)$$

- TFP growth comes from
  - Improvements in productivity on tasks that don't switch automation status
  - Automation effect (switching tasks from labor to capital)
  - Other TFP growth ( $\hat{Z}_t$ )

## Automation Indifference Condition

- Setting
  - **Continuity:** Let  $\psi_{kit}$ ,  $\psi_{lit}$ ,  $w_t$ , and  $r_t$  only change smoothly
  - Any task  $\beta$  that is just at the margin of being automated:  $\frac{\psi_{k\beta t}}{\psi_{l\beta t}} = \frac{r_t}{w_t}$
- Therefore, at marginal task  $\beta$ :

$$\begin{aligned} s_{Lt}\omega_{l\beta t} - s_{Kt}\omega_{k\beta t} &= \frac{w_t L_t}{P_t Y_t} \cdot \frac{w_t L_{\beta t}}{w_t L_t} - \frac{r_t K_t}{P_t Y_t} \cdot \frac{r_t K_{\beta t}}{r_t K_t} \\ &= \frac{w_t L_{\beta t}}{P_t Y_t} - \frac{r_t K_{\beta t}}{P_t Y_t} = 0 \end{aligned}$$

- **Automation effect is zero!**
  - Automation occurs precisely where costs of using labor and capital are equal
  - Analogy to entry in firm dynamics literature

### Proposition 3 (*Zero TFP growth at instant of automation*)

The automation effect in the TFP growth decomposition (4) is zero. Therefore, TFP growth equals

$$\begin{aligned}\widehat{TFP}_t &= s_{Kt}\hat{B}_t + s_{Lt}\hat{A}_t \\ &= \underbrace{s_{Kt}\hat{\psi}_{kt}}_{\text{Capital productivity}} + \underbrace{s_{Lt}\hat{\psi}_{\ell t}}_{\text{Labor productivity}} + \underbrace{\hat{Z}_t}_{\text{Other productivity}}\end{aligned}$$



How much of TFP growth is due to  
better machines:  $\hat{\psi}_{kt}$ ?

## Measuring Contribution of Automation to Growth

$$\widehat{TFP}_t = \underbrace{\hat{Z}_t + \hat{\psi}_{\ell t}}_{\text{Baseline TFP growth}} + s_{Kt} (\hat{\psi}_{kt} - \hat{\psi}_{\ell t}) \quad (5)$$

Automation boost  
from machines  
getting better

- Rearrange growth into a baseline growth rate plus an automation boost
- Two key objects to measure:  $\hat{Z}_t + \hat{\psi}_{\ell t}$  and  $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$

## Measuring Contribution of $\hat{Z}_t + \hat{\psi}_{\ell t}$

- Recall, labor augmenting productivity growth (from Proposition 2):

$$\hat{A}_t = \hat{Z}_t + \hat{\psi}_{\ell t} + \frac{1}{1 - \sigma} \bar{\omega}_{\ell\beta t} \dot{\beta}_t \quad (6)$$

### Definition 1

The *automation rate*,  $x_t$ , is defined as the share of total labor costs accounted for by newly-automated tasks before they were automated:

$$x_t := \bar{\omega}_{\ell\beta t} \dot{\beta}_t = \sum_{m=1}^{M_t} \omega_{\ell\beta_{mt}} \dot{\beta}_{mt}. \quad (7)$$

## Measuring Contribution of $\hat{Z}_t + \hat{\psi}_{\ell t}$

### Proposition 4 (*Identifying $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$ and $\hat{Z}_t + \hat{\psi}_{\ell t}$* )

If we measure the automation rate  $x_t$ , then  $\hat{Z}_t + \hat{\psi}_{\ell t}$  and  $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$  are identified:

$$\hat{Z}_t + \hat{\psi}_{\ell t} = \hat{A}_t - \frac{1}{1 - \sigma} x_t,$$

$$\hat{\psi}_{kt} - \hat{\psi}_{\ell t} = \frac{1}{s_{Kt}} \left( \widehat{TFP}_t - (\hat{Z}_t + \hat{\psi}_{\ell t}) \right).$$

## Measurement in Practice: Task Categories

- Partition the unit interval into 100 discrete task categories  $J_j$ , indexed by  $j$
- Define the automation rate at the task category level:

$$x_t^j := \sum_{m \in J_j} \omega_{\ell\beta_{mt}}^j \dot{\beta}_{mt}, \quad (8)$$

$\omega_{\ell\beta_{mt}}^j \equiv \frac{L_{\beta_{mt}}^j}{L_t^j}$  is share of labor costs in category  $j$  used on just-automated task  $\beta_{mt}$

- Overall automation rate is simply the weighted sum of category automation rates. Weights,  $\tilde{\omega}_{\ell t}^j$ , are the shares of total labor costs accounted for by category  $j$ :

$$x_t = \sum_j \tilde{\omega}_{\ell t}^j x_t^j \quad (9)$$

- We proceed to measure  $\tilde{\omega}_{\ell t}^j$  and  $x_t^j$  to compute  $x_t$

## Identification of $\beta_t$ through Narrative Historical Analysis

- 1) For each industry, we ask a **world-class specialist historian** to list 100 specific task categories that are necessary for production of that industry
    - Consider years 1987, 1997, 2007, and 2017 (longer for Agriculture and Pvt Bus)
  - 2) For each task category-year, what fraction of tasks were automated:  $\beta_t^j$
  - 3) For each task category-year, what is share of total labor costs were accounted for by category  $j$ :  $\tilde{\omega}_{\ell t}^j$
- Assume that labor cost shares within a task-category are equal, which implies that  $\omega_{\ell, \beta_{mt}}^j \equiv \frac{L_{\beta_{mt}}}{L_t^j} = \frac{1}{1 - \beta_t^j}$  (weights across categories are not quantitatively important)
  - Assumption, plus measured  $\beta_t^j$  and  $\tilde{\omega}_{\ell t}^j$  and equations (8) and (9) yield  $x_t$
  - Historian is ChatGPT and Claude LLMs. Repeat 25 times each and average

## Most and Least Automated Task Categories

### Least Automated Tasks

 $\beta$ 

### Most Automated Tasks

 $\beta$ 

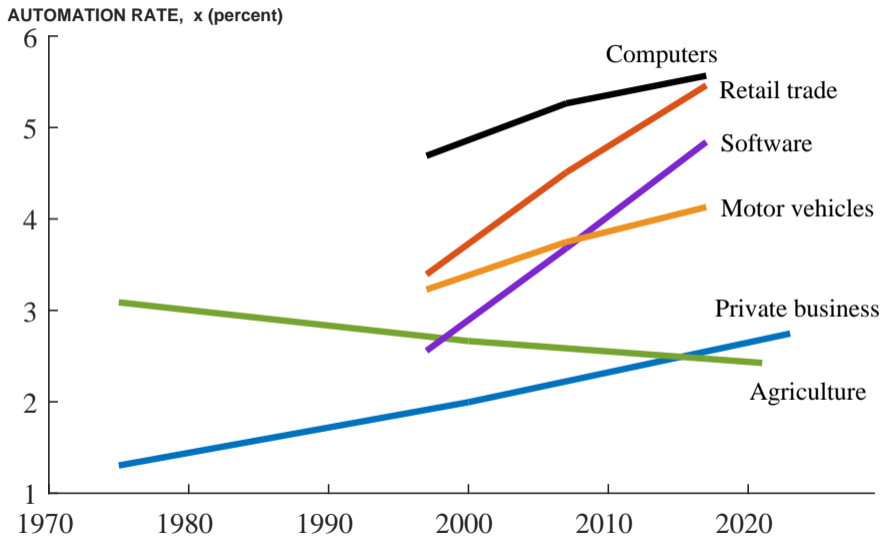
#### *Private Business Sector, 2023*

Graft or bud fruit trees	0.159	Thresh harvested grain	0.993
Classroom instruction	0.207	Harvest small grains (reap/bind)	0.991
Administer vaccinations	0.219	Execute interbank funds transfer	0.989
Assist at surgery	0.220	Grain milling	0.989
Basic wound care and dressing	0.220	Textile spinning	0.989

#### *Motor Vehicles, 2017*

Route and secure cables/hoses	0.174	Cylinder bore machining/honing	0.981
Install exterior trim and emblems	0.187	Phosphate conversion coating	0.979
Install headliner and pillar trims	0.195	Cylinder head machining	0.977
Lay carpets and sound-deadening mats	0.197	Transmission gear hobbing & heat treat	0.976
Install door hardware and locks	0.213	Body cleaning and degreasing	0.976

## Automation Rate, $x_t$



## Top 5 Task Contributors to the Automation Rate

Task category	Labor cost share, $\bar{\omega}_j$	Automation rate, $x_j$	Contribution of task $j$	Share
<i>Agriculture, 2000–2021</i> ( $x = 2.4\%$ )				
Feed livestock	0.054	0.037	0.0019	7.9%
Milk dairy animals	0.044	0.042	0.0018	7.6%
Hand-pick fruits/vegetables	0.144	0.008	0.0012	4.8%
Collect & handle eggs	0.020	0.048	0.0010	4.3%
Spray/dust pesticides	0.017	0.041	0.0007	2.8%
				Top 5 tasks account for 27.3%
<i>Retail Trade, 2007–2017</i> ( $x = 5.5\%$ )				
Process non-cash payments	0.027	0.110	0.0029	5.3%
Compute totals and tax	0.017	0.131	0.0023	4.2%
Sales/labor/inventory reports	0.020	0.108	0.0021	3.9%
Gasoline-station courtesy service	0.027	0.067	0.0020	3.7%
Schedule and timekeeping	0.022	0.088	0.0019	3.5%
				Top 5 tasks account for 20.5%

## Automation Boost $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$ Drives Growth

Sector	Growth rate of $A_t$	Automation rate, $x_t$	Baseline $\hat{Z}_t + \hat{\psi}_{\ell t}$	TFP Growth	Capital share, $s_{Kt}$	Automation boost, $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$
<i>Panel A: Main Results</i>						
Private business	2.3	2.0	-0.2	1.2	0.35	3.8
Agriculture	4.3	2.7	0.9	3.3	0.57	4.4
Computers	14.9	5.2	8.4	12.8	0.41	10.7
Motor vehicles	2.7	3.7	-2.0	1.7	0.43	8.6
Retail trade	2.4	4.5	-3.1	1.7	0.20	23.3
Software	4.6	3.7	-0.1	1.8	0.47	4.0

$\hat{Z}_t + \hat{\psi}_{\ell t}$  is typically small or negative

$\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$  is at least 4% per year

## Robustness to Mismeasured TFP and Automation Rate $x_t$

Sector	Growth rate of $A_t$	Automation rate, $x_t$	Baseline $\hat{Z}_t + \hat{\psi}_{\ell t}$	TFP Growth	Capital share, $s_{Kt}$	Automation boost, $\hat{\psi}_{Kt} - \hat{\psi}_{\ell t}$
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### *Panel B: Robustness to Mismeasurement of $x_t$*

Private business	2.3	1.8	0.0	1.2	0.35	3.3
Agriculture	4.3	2.7	0.9	3.3	0.57	4.4
Computers	14.9	5.2	8.4	12.8	0.41	10.7
Motor vehicles	2.7	2.1	0.0	1.7	0.43	4.1
Retail trade	2.4	1.9	0.0	1.7	0.20	8.2
Software	4.6	3.6	0.0	1.8	0.47	3.9

### *Panel C: Robustness to Mismeasurement of TFP Growth*

Private business	2.5	2.0	0.0	1.4	0.35	3.8
Agriculture	4.3	2.7	0.9	3.3	0.57	4.4
Computers	14.9	5.2	8.4	12.8	0.41	10.7
Motor vehicles	4.6	3.7	0.0	3.7	0.43	8.6
Retail trade	5.6	4.5	0.0	4.8	0.20	23.3
Software	4.6	3.7	0.0	1.9	0.47	4.0



How much TFP growth would be lost  
if we “froze” automation in 1950 (or 1987)?

## “Freeze” Automation in 1950 or 1987

- Counterfactual accounting exercise, using historical data
- Consider the set of automated tasks from some initial year
  - Hold automation set fixed at that initial value:  $\Omega_{kt} = \Omega_{k0}$
  - Assumption 3:  $\hat{\psi}_{kt}$ ,  $\hat{\psi}_{lt}$ , and  $\hat{Z}_t$  are unchanged.
- Recall TFP growth:

$$\widehat{TFP}_t = s_{Kt}(\hat{\psi}_{kt} - \hat{\psi}_{lt}) + \hat{Z}_t + \hat{\psi}_{lt}$$

→ only affects TFP growth via  $s_{Kt}$

## (continued)

- Hold the set of tasks fixed while machines continue to get better:
  - Recall:  $\uparrow\beta_t \Rightarrow \uparrow s_{Kt}$  and  $\uparrow\psi_{kt} \Rightarrow \downarrow s_{Kt}$
  - Holding  $\beta$  fixed  $\Rightarrow$  counterfactual capital share will be **lower**:

$$\frac{s_{Kt}^{cf}}{1 - s_{Kt}^{cf}} = \frac{s_{Kt}}{1 - s_{Kt}} \exp\left(-\int_0^t \frac{1}{s_{K\tau}} x_\tau d\tau\right) \quad (10)$$

- Lower share of costs  $s_{Kt}$  benefits from  $\hat{\psi}_{kt} - \hat{\psi}_{\ell t} \Rightarrow$  “lost” TFP growth

### Proposition 5 (*Counterfactual contribution of automation*)

Under Assumptions 1 and 2, the lost TFP growth from “freezing” the set of automated tasks in some historical year satisfies

$$\widehat{TFP}_t - \widehat{TFP}_t^{cf} = (s_{Kt} - s_{Kt}^{cf}) (\hat{\psi}_{kt} - \hat{\psi}_{\ell t}),$$

where  $s_{Kt}^{cf}$  is given by equation (10) and  $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$  is given in Proposition 4.

## Counterfactual Lost Growth from Freezing Automation

Sector	Capital share		$\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$	Lost Growth $\widehat{TFP}_t - \widehat{TFP}_t^{cf}$	TFP Growth	Lost Growth Share of $\widehat{TFP}_t$
	$s_{K,T}$	$s_{K,T}^{cf}$				
<i>Automation set frozen in 1950:</i>						
Private business	0.420	0.000	3.8	1.3	1.2	108%
Agriculture	0.655	0.052	4.4	1.5	3.3	45%
<i>Automation set frozen in 1987:</i>						
Computers	0.459	0.018	10.7	3.2	12.8	25%
Motor vehicles	0.524	0.073	8.6	2.2	1.7	126%
Retail trade	0.259	0.000	23.3	4.1	1.7	242%
Software	0.463	0.076	4.0	1.0	1.8	53%

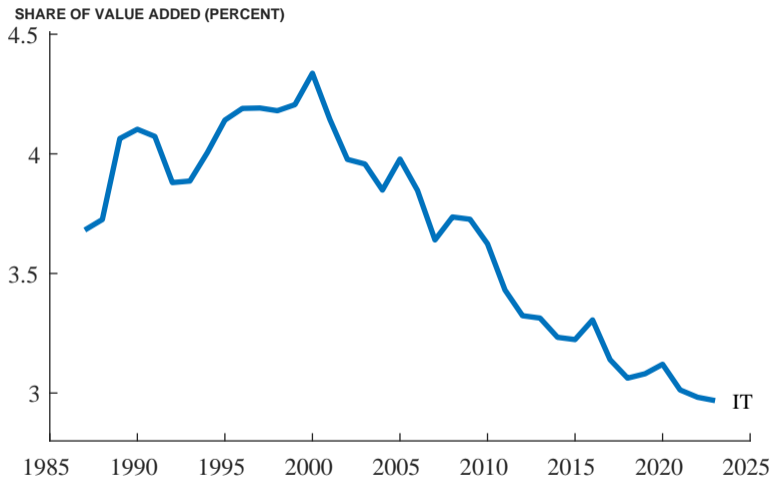
*Rapid machine productivity improvement not enough  
Need expanding set of tasks for automation to drive growth*



## The Future Consequences of A.I.?

## What has happened to the factor income share of computers / IT?

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*Computers are everywhere, but rapid price declines dominate.  $\sigma < 1$*

## The Role of Weak Links ( $\sigma < 1$ )

- Firm production requires the successful completion of a number of **tasks**
  - Failing at sourcing inputs or quality control or timely delivery or other tasks can be very detrimental
  - Examples: the space shuttle Challenger's O-ring or Covid-19 supply chain issues
- Automation  $\Rightarrow$  fast computers / machines perform tasks instead of people
  - Large cost savings in long run — machines get better rapidly
  - Talented people are the scarce input

*Weak links remain the limit to production and hence earn high returns*

## What are the consequences of fully automating software?

- A simple warm-up calculation is revealing
  - Assume  $\psi_{kit}$  on the fully-automated tasks becomes **infinite**
  - Assume the set of tasks that benefit is frozen, and no effect on idea production
  - We relax these assumptions later
- $\Omega_\infty$  = set of infinitely automated tasks,  $\Omega_\emptyset$  = other tasks (unaffected)
- Before infinite automation,

$$\begin{aligned} Y_t^{\frac{\sigma-1}{\sigma}} &= \int_0^1 Y_{it}^{\frac{\sigma-1}{\sigma}} di \\ &= \int_{\Omega_\infty} Y_{it}^{\frac{\sigma-1}{\sigma}} di + \int_{\Omega_\emptyset} Y_{it}^{\frac{\sigma-1}{\sigma}} di \\ &= Y_{\infty t}^{\frac{\sigma-1}{\sigma}} + Y_{\emptyset t}^{\frac{\sigma-1}{\sigma}} \end{aligned}$$

## The Future of Automation: Perfectly Automated Software

- Today's **pre-automation** shares:

$$s_{jt} \equiv \frac{P_{jt}Y_{jt}}{P_tY_t} = \left(\frac{Y_{jt}}{Y_t}\right)^{\frac{\sigma-1}{\sigma}}$$

- With infinite  $\psi_k$ ,  $Y_\infty$  to infinity, so  $Y_\infty^{\frac{\sigma-1}{\sigma}}$  to zero — because  $\sigma < 1$

$$Y_{cf} = Y_{\emptyset t}$$

- We essentially removed some of the **weak links**

## The Future of Automation: Perfectly Automated Software

- Dividing by  $Y_t$ ,

$$\begin{aligned}\frac{Y_{cf}}{Y_t} &= \frac{Y_{\emptyset t}}{Y_t} \\ &= s_{\emptyset t}^{\frac{\sigma}{\sigma-1}}\end{aligned}$$

$$\frac{Y_{cf}}{Y_t} = \left( \frac{1}{1-s_{\infty}} \right)^{\frac{\sigma}{1-\sigma}} \approx 1 + \frac{\sigma}{1-\sigma} s_{\infty} \quad \text{for } s \text{ small} \quad (11)$$

- Key insight:
  - $s \approx 2\%$  in software  $\Rightarrow$  about 0.5% increase in GDP from complete-automation of the software industry with infinite productivity!
  - 11% increase from complete automation of tasks that currently cost 1/3 of GDP (e.g., cognitive labor)

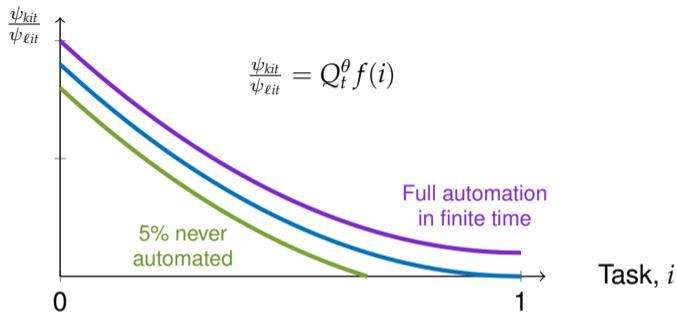
## The Dynamic Model including Automating Idea Production

- The model so far continues to apply  $\Rightarrow Y_t$  and  $\Omega_{kt}$ , etc.
- Dynamics via the production of new ideas
  - Ideas growth  $\Rightarrow \psi_{kit}$  and  $\psi_{elit}$  growth
  - Lab equipment setup: automating goods also automates ideas
- Calibrate model to match the facts we've already documented
- Use the model to project the future
  - What happens to growth and factor shares as automation continues?

## The Dynamic Model: Semi-endogenous growth plus automation

CES task model	Same as before $\Rightarrow Y_t$ and $\Omega_{kt}$
Idea PF	$\dot{Q}_t = \bar{q}R_t^\lambda Q_t^\phi$
Ideas $\Rightarrow \psi_{kit}$	$\psi_{kit} = Q_t^{\theta_k} f(i)$
Ideas $\Rightarrow \psi_{\ell it}$	$\psi_{\ell it} = Q_t^{\theta_\ell}$ (homogeneous)
Heterogeneity	$f(i) = \frac{(1-i)^\mu}{1+\mu_0(1-i)^\mu} + \bar{f}$
Resource constraint	$C_t + I_t + R_t = Y_t$
Capital accumulation	$\dot{K}_t = I_t - \delta K_t$
Population growth	$L_t = L_0 e^{nt}$
Allocations	$R_t = \bar{u}_R Y_t$ and $I_t = \bar{u}_K Y_t$

## Automation in the Long Run



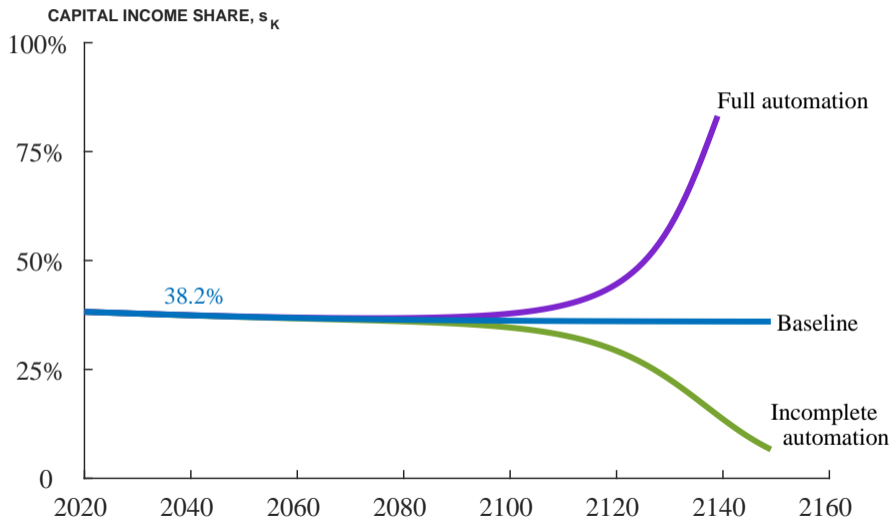
- Some tasks **never automated** if  $f(\beta) = 0$  for  $\beta > \beta^*$
- All tasks **automated in finite time** if  $f(1) > 0$
- What happens if  $f(1) = 0$ ?
  - Labor always used, but on a vanishing range of tasks

## Calibrating to Historical Evidence (Private Business Sector)

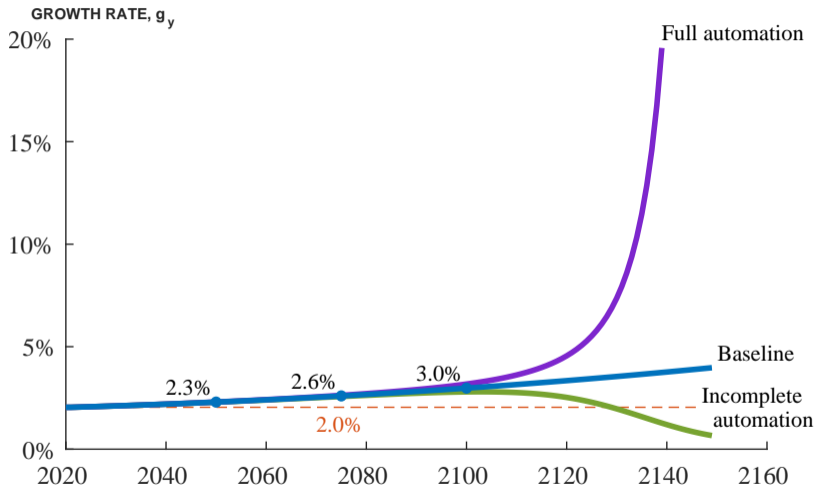
<u>Moment or Parameter</u>	<u>Value</u>	<u>Source</u>
<i>Moments from first half of the paper</i>		
Capital shares, $s_{Kt}$	.35, .33, .33, .42	1950, 1975, 2000, 2023
Automation cutoffs, $\beta_t$	.28, .45, .63, .75	1950, 1975, 2000, 2023
Task TFP growth, $\hat{\psi}_k - \hat{\psi}_\ell$	3.8% or 10.7%	Pvt Business or Computers
Labor task TFP growth, $\hat{\psi}_\ell$	0.5%	Assume humans improve slowly
“Other” TFP growth, $\hat{Z}_t$	-0.7%	From $\hat{A}_t$ and automation rate. Decays to zero

- Two sets of simulations
  1. A.I. as a continuation of historical patterns = Private Business Sector
  2. A.I. as a break with past: Moore's Law everywhere = calibrate to Computer sector

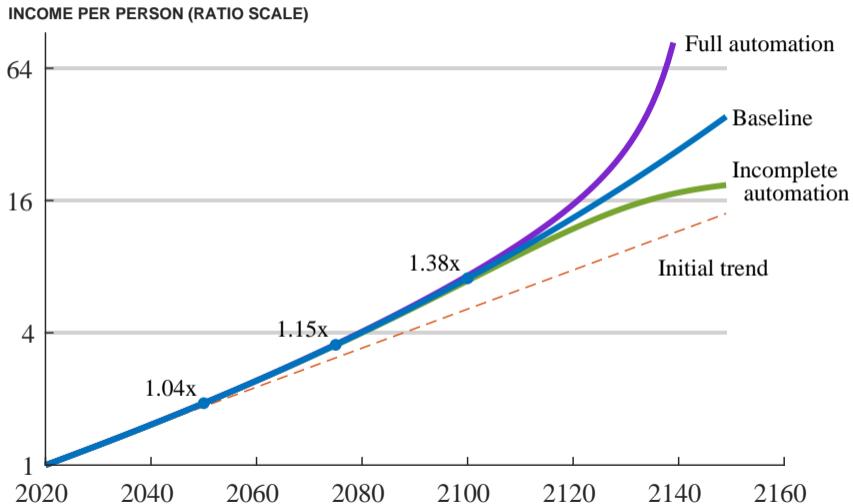
## The Future if AI is Continuation of Past: Capital Share



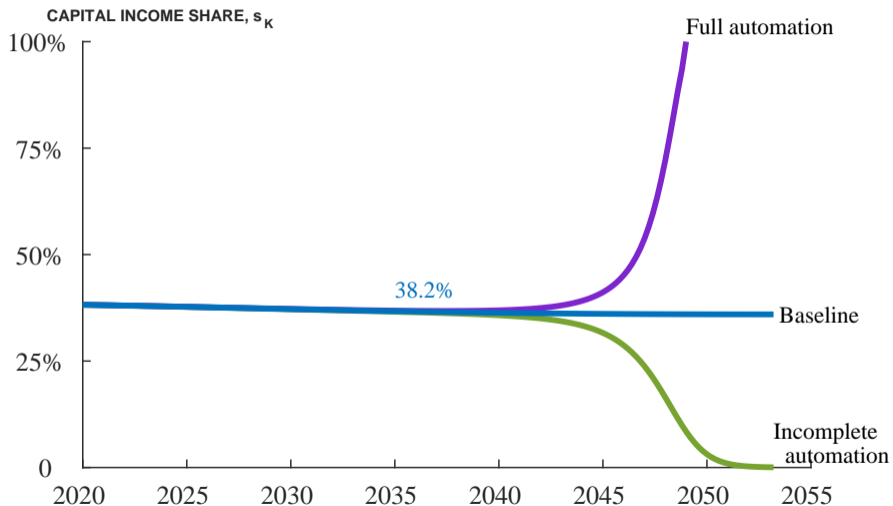
## The Future if AI is Continuation of Past: Economic Growth



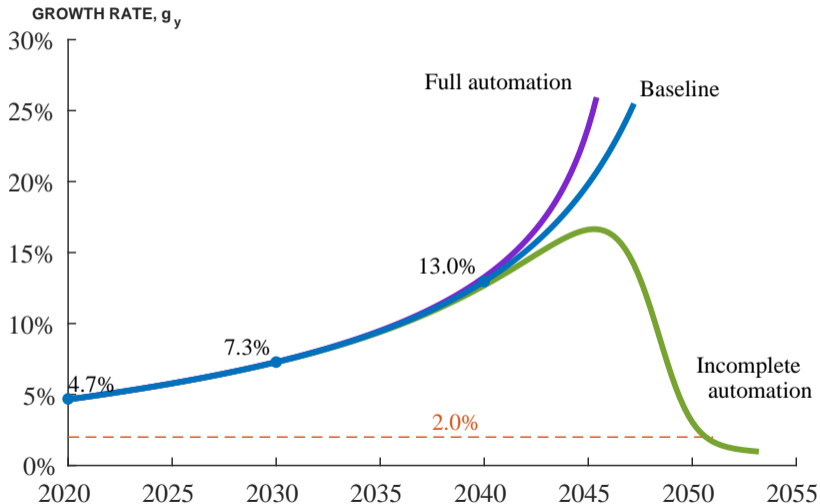
## The Future if AI is Continuation of Past: GDP per Person



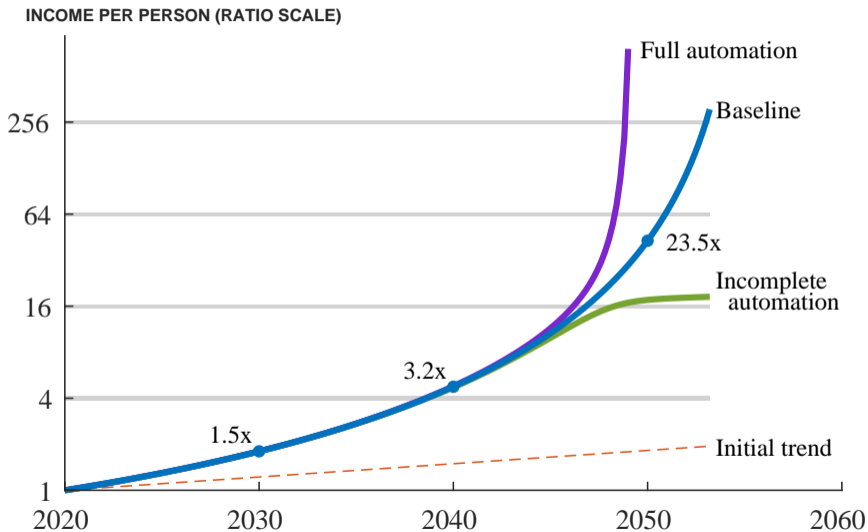
## The Future if AI = Computers for Aggregate: Capital Share



## The Future if AI = Computers for Aggregate: Economic Growth



## The Future if AI = Computers for Aggregate: GDP per Person



## Summary: The Future of Growth

- In LR: 100% capital share, 100% labor share, or stable 2/3 labor share all possible
  - depends on if machines perform all tasks, humans do some tasks forever, or humans do a vanishing share of tasks
- For next 20 years, these very different LR outcomes are indistinguishable
- Growth can explode, but even explosion is slow (decades)
- Explosion depends on whether AI is different from past, eliminating weak links faster

## Intuition for Slowly Accelerating Growth

- With  $\bar{f} = 0$ , the capital share stabilizes at  $s_K^* = \frac{1}{\mu(1-\sigma)}$  (weak links)
- Key measure of dynamic increasing returns:

$$\Phi = \frac{\lambda}{1-\phi} \left( \theta_\ell + \frac{s_K^*}{1-s_K^*} \theta_k \right)$$

- When  $\Phi < 1 \Rightarrow$  semi-endogenous growth:

$$g_y = \frac{\Phi n}{1-\Phi}$$

- $\Phi = 1 \Rightarrow$  knife edge of fully endogenous growth
- $\Phi > 1 \Rightarrow$  explosive growth

## Intuition (continued)

- With our baseline parameter values:  $\Phi_{pbs} = 0.98$  and  $\Phi_{computers} = 2.27 \Rightarrow$  **explodes**
  - Private Business: SEG but LR growth rate is  $\frac{\Phi n}{1-\Phi} = 47\%$
- Mapping  $\Phi$  back to moments in the data:

$$\Phi \approx \frac{\hat{\psi}_{et} + \frac{1}{1-\sigma} x_0}{gY_0}$$

Weak links:  $\downarrow \sigma \Rightarrow \downarrow \Phi$

Automation rate:  $\uparrow x \Rightarrow \uparrow \Phi$

- Our cases:
  - Private Business:  $\Phi \approx \frac{0.5\% + 1.25 \cdot 2.0\%}{3\%} \approx 1$
  - Computers:  $\Phi \approx \frac{0.5\% + 1.25 \cdot 5.2\%}{3\%} \approx 2.33$

## Speed of Transition Dynamics

- Weak links slow the rate at which the growth rate increases
- Business as usual (Private Business Sector based calibration)
  - Start with flat growth at 2% per year
  - Transiting to 47% growth  $\Rightarrow$  takes a long time
- A.I. as a break with the past (Computers based calibration)
  - Consider a one-dimensional example:  $\dot{X}_t = \bar{g}X_t^\Phi$  . The growth rate is  $\hat{X}_t = \bar{g}X_t^{\Phi-1}$   
 $\Rightarrow$  explodes for  $\Phi > 1$
  - Integrating: Infinite income occurs in finite time at date

$$t_\infty = \frac{1}{(\Phi - 1)\bar{g}} = \frac{1}{1.27 \times 0.047} = 17 \text{ years}$$

## Conclusion

- How much of historical growth is due to automation?
  - More than 50%
  - The gain from automation is switching from slowly-improving humans (e.g., 0.5% per year) to rapidly-improving machines (e.g., 4% per year) on ↑ share of tasks
- The future of A.I.?
  - Weak links are crucial
  - Having infinite computing power on a subset of tasks is still limited by weak links
  - Automation could certainly accelerate growth
    - ... but the effects may take decades to be substantial

## Appendix Slides

## Robustness

Sector	$\sigma = 0.1$	$\sigma = 0.3$	$\sigma = 0.5$	$x_t \rightarrow 0.8x_t$	GPT-5.5	Claude Opus 4.7
"Other" TFP growth: $\hat{Z}_t + \hat{\psi}_{\ell t}$						
Private business	0.1	-0.5	-1.6	0.3	0.0	-0.4
Agriculture	1.2	0.4	-0.9	1.6	1.4	0.3
Computers	9.0	7.7	5.2	9.7	9.0	7.8
Motor vehicles	-1.6	-2.4	-3.9	-1.0	-1.1	-2.8
Retail trade	-2.6	-3.8	-6.1	-2.0	-2.8	-3.5
Software	0.4	-0.7	-2.6	0.9	0.3	-0.4
Excess rate at which machines getting better: $\hat{\psi}_{kt} - \hat{\psi}_{\ell t}$						
Private business	3.1	4.8	7.8	2.4	3.3	4.4
Agriculture	3.8	5.1	7.5	3.2	3.4	5.3
Computers	9.3	12.6	18.4	7.6	9.3	12.1
Motor vehicles	7.8	9.7	13.0	6.4	6.6	10.6
Retail trade	20.6	26.7	37.5	17.9	21.7	24.9
Software	3.0	5.3	9.4	2.1	3.3	4.7

## Robustness

Sector	$\sigma = 0.1$	$\sigma = 0.3$	$\sigma = 0.5$	$x_t \rightarrow 0.8x_t$	GPT-5.5	Claude Opus 4.7
Lost TFP growth – freeze automation (percent share)						
Private business	91	129	190	70	96	121
Agriculture	40	52	73	30	33	59
Computers	21	29	43	16	21	29
Motor vehicles	114	141	190	83	86	168
Retail trade	214	278	393	179	220	264
Software	39	71	128	24	40	66